Lesson 9-3: Angles of Elevation and Depression

Depressed angles?

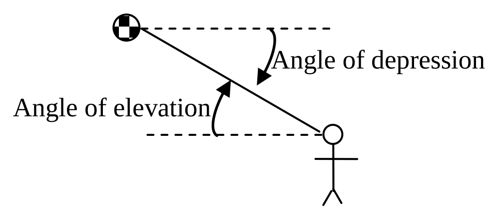
Can an angle be depressed? How about ecstatic? The answer is yes and no. Yes, angles can be "depressed." No, angles can not be ecstatic, but they can be "elevated." Today's lesson gives you a bit of an exposure to the math that a surveyor uses.

Angles of depression and elevation

Suppose while we're playing soccer, the keeper kicks the ball down field and we're able to freeze the ball mid-flight. All of the players (well, any that are paying attention!) would be looking up at the ball in the air. A valid question to ask is: what angle is the ball up in the air? To answer that we need to be a bit more precise; we'd likely add "from the ground." I think we'd all agree this addition is helpful. But it isn't enough. The angle of the ball from ground depends on where you're standing doesn't it? The center is likely much closer to the ball by now than the keeper that kicked the ball. The angle from the center to the ball would be different than the angle from the keeper to the ball. So we should change our question to "what is the angle from the center to the ball?" There, that is precise enough to work with!

This is called the <u>angle of elevation</u>: the ball is "elevated" above the player. It is measured from a horizontal line through the view point (the player) *up* to the object (ball).

Now I suppose you could ask the question the other way around: "what is the angle from the ball to the player?" I'm not sure that is a very interesting question to ask but it is valid non-the-less.



This is called the <u>angle of depression</u>: the player is below the ball. Think of a depression in the ground; it is below the ground surface. It is measured from a horizontal line through the view point (the ball) *down* to the object (player).

<u>It is important to note</u> that both angles are from horizontal lines. In this diagram, the dashed lines are horizontal. If both lines are horizontal, what can you say about how they relate to one another? They are parallel to one another. How then could you classify the angle of elevation and angle of depression? They are alternate interior angles with the line-of-sight between the player and the ball being the transversal line. Hence they are congruent angles.

Identifying angles of elevation and depression

As we've noted, you need to be careful and precise when you describe angles of elevation or depression. An angle of elevation or depression is a "from-to" relationship. It is an angle measured from one thing to another. Using the soccer diagram above we have:

1. The angle of elevation <u>*from*</u> the player <u>*to*</u> the ball.
2. The angle of depression <u>*from*</u> the ball <u>*to*</u> the player.

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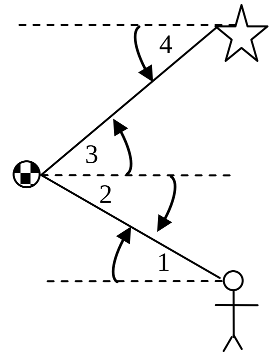
Examples



1. Describe each angle as it relates to the situation shown.

   $\angle 1$ is the angle of elevation from the player to the ball.

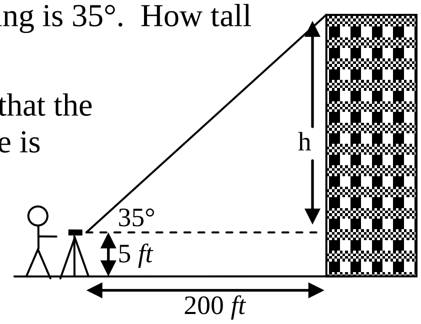
   $\angle 2$ is the angle of depression from the ball to the player.

   $\angle 3$ is the angle of elevation from the ball to the star.

   $\angle 4$ is the angle of depression from the star to the ball.

   Notice how each starts ***from*** the vertex of the angle and goes ***to*** the viewed object. As you will see in the next example, it may not be stated explicitly like this. However, this *from-to* relationship is ***always*** expressed.

2. A surveyor stands 200 *ft* from a building to measure its height with a 5 *ft* tall theodolite. The angle of elevation to the top of the building is 35°. How tall is the building?



   First, draw the situation: it is important to remember that the theodolite is 5 *ft* off the ground. This means the angle is measured from a point 5 *ft* off the ground.

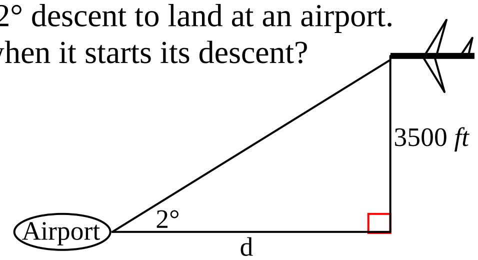
   Next, what information do we have? We know the angle measure and the length of the adjacent side of the triangle we've formed.

   What do we want to find? The opposite side of the triangle; this means we'll used the tangent ratio.

   $$\tan 35 = \frac{h}{200};\ h = 200\tan 35 = 140.04 \approx 140\,ft$$

   Now, we can't forget that the measuring device (theodolite) is 5 *ft* off the ground. The height of the building then is h + 5 *ft* or 140 + 5 = 145 *ft*.

3. An airplane flying 3500 *ft* above ground begins a 2° descent to land at an airport. How many miles from the airport is the airplane when it starts its descent? (note: 1 mile = 5280 *ft*).



   Again, draw it out first: we are looking for *d*.

   Since we know the angle measure and the opposite side, and we're looking for the adjacent side, we will use the tangent ratio.

   $$\tan 2° = \frac{3500}{d};\ d = \frac{3500}{\tan 2°} = 100{,}226.88 = 18.98\,miles \approx 19\,miles$$

Homework Assignment

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